

Quantitative Estimation Of Wake Vortex Safety Using The P2P Model

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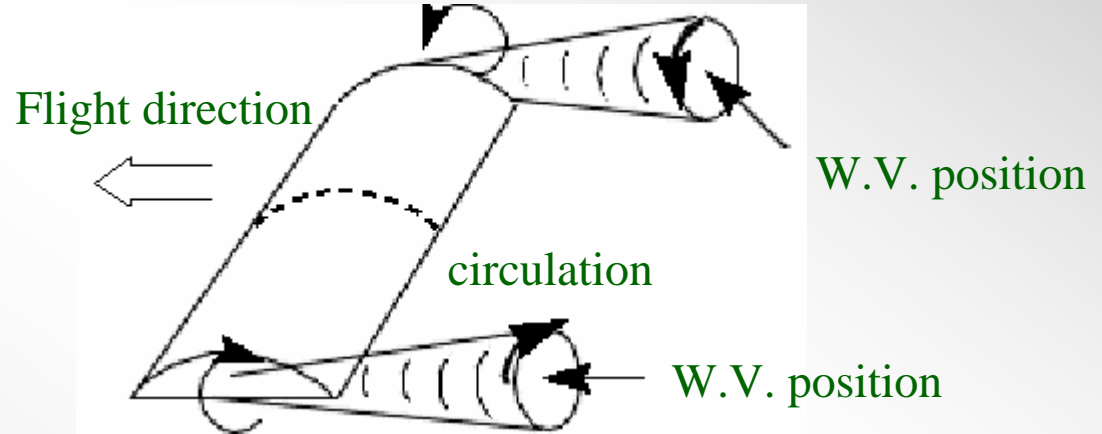
Wake Vortex



Wake vortices are generated by *the difference* between the low pressure on the top surface and the high pressure on the bottom surface of a wing.



NASA Langley



Subject to:

acft weight, wing span, wing aspect ratio, etc;
air density, viscosity, temperature, stratification;
wind speed, direction

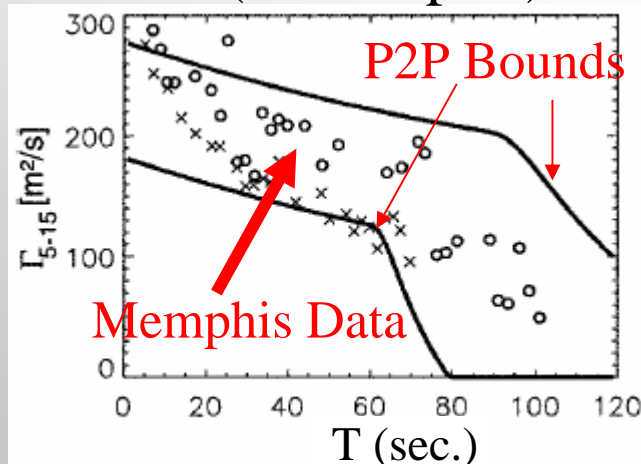
P2P (Prob. 2-Phase) WV Model



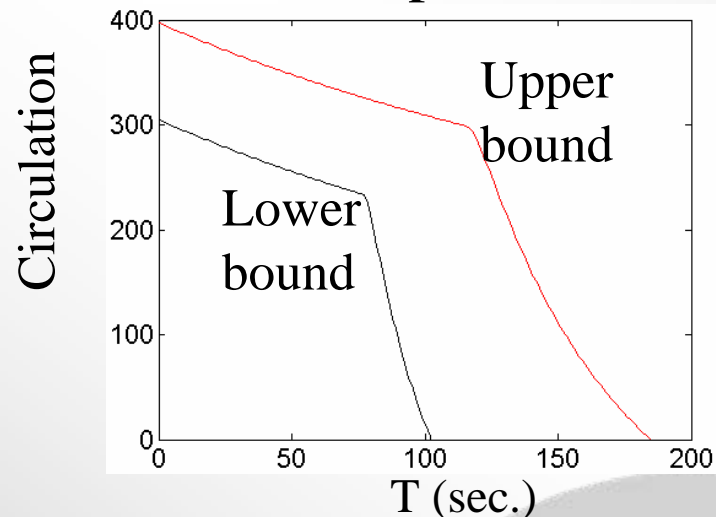
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- Use Γ_{5-15} to describe circulation
- Based on a solution to Navier-Stokers equ.
- Two-phase wake decay
- Adjustable bounds to cover uncertainties
- Compared with Memphis WV data

P2P (Holzaepfel)



P2P (GMU Implementation)



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Holzaepfel, F. (2003). *Probabilistic Two-Phase Wake Vortex Decay and Transport Model*. *Journal of Aircraft*, Vol. 40, No. 2, March ~ April 2003.



Evaluate WV Encounter Risk

Define events

A = a trailing acft $j+1$ passes X during $[T+\Delta t/2, T-\Delta t/2]$ after acft j . $T \in [T_0, T_1]$

B = WV stronger than A at age T ; by default, $A=0$

C = WV and acft $j+1$ overlap in Z dimension at X , at T

D = WV and acft $j+1$ overlap in Y dimension at X , at T

Define $I(j, X, T) = \begin{cases} 1, & \text{if } A \cap B \cap C \cap D \\ 0, & \text{otherwise} \end{cases}$

$$P(I(j, X, T) = 1) = P(A \cap B \cap C \cap D)$$

$$= P(CD \mid AB) \times P(AB)$$

Assume WV decay is indep. of trailing acft's passage

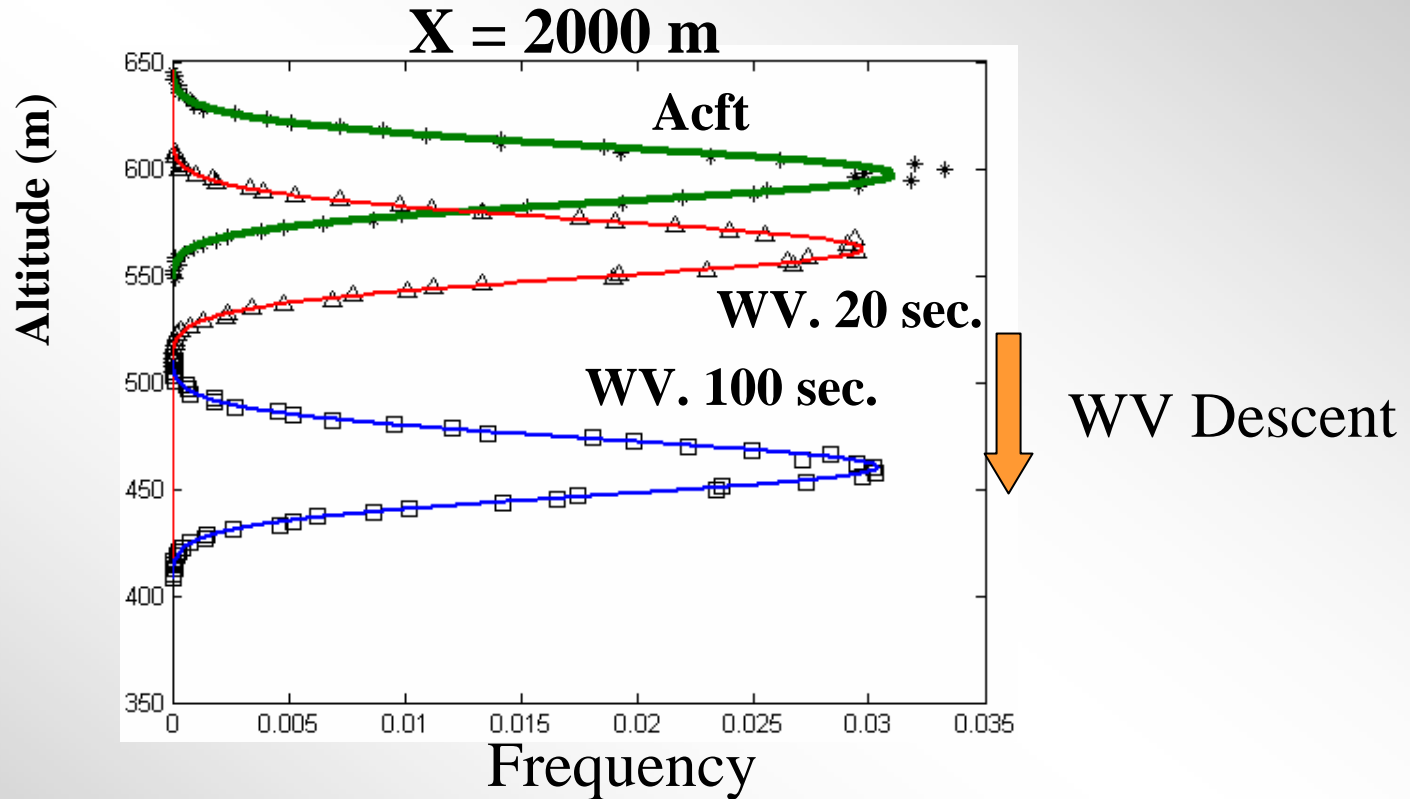
$$= \underline{P(C \mid AB)} \times \underline{P(D \mid AB)} \times P(A) \times P(B)$$

Assume overlappings in latitude and altitude are independent

Overlapping Prob. in Altitude



$$\underline{P(C|AB) * P(D|AB) * P(A) * P(B)}$$



$P(C | AB)$ is the altitudinal overlapping probability of Acft and WV

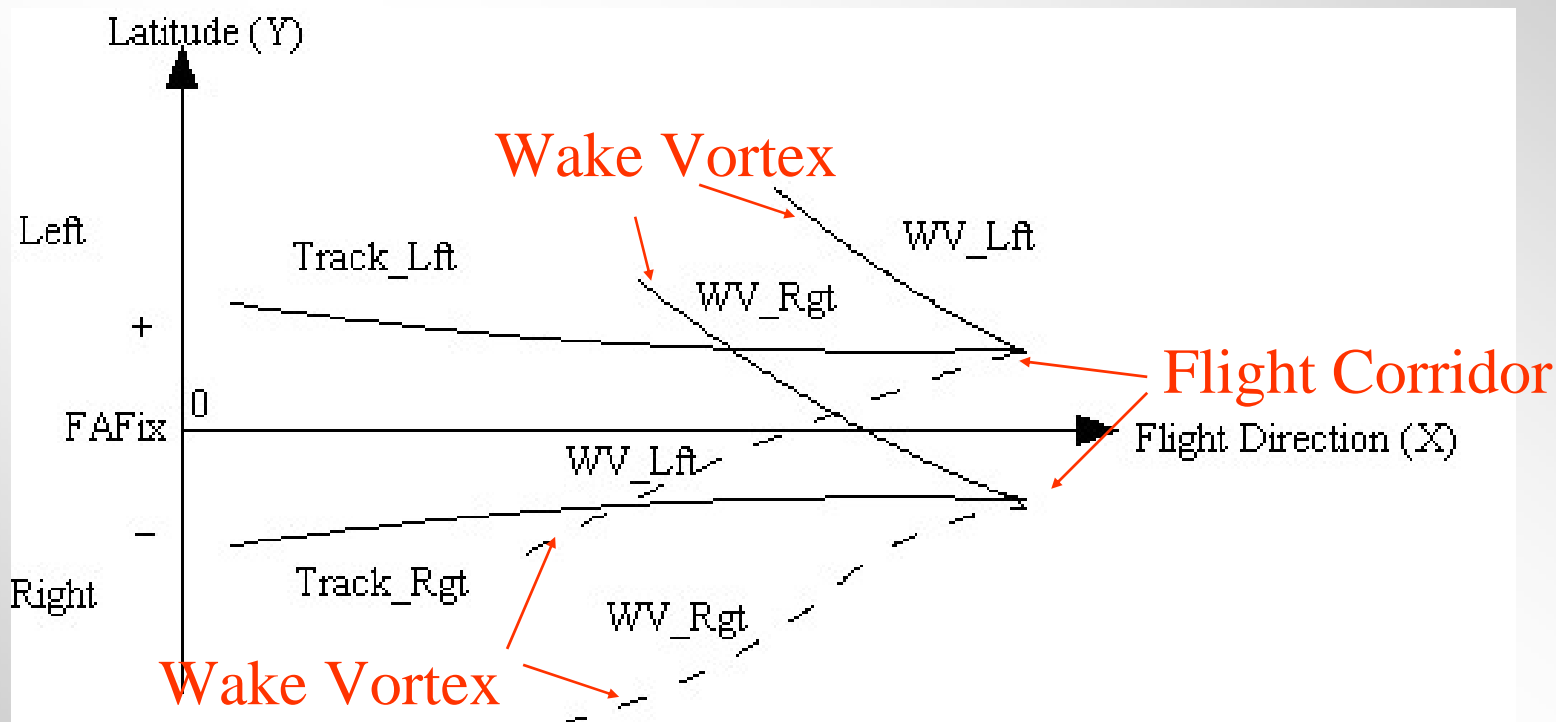
(Conditional) Probability: 0.032 Vs. $6 \cdot 10^{-14}$

Overlapping Prob. in Lateral Direction



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$$\underline{P(C|AB) * P(D|AB) * P(A) * P(B)}$$



$P(D|AB) \cong P(\text{either trail of WV stays in flight corridor})$

Flight corridor is determined by aircraft wing span
+ lateral deviance

Aircraft Separation Distribution



- Acft Flight Model:
- WV sep. enforced at Final Approach Fix (FAF), w/ variance
 - Acft fly randomly picking but constant (ground) speed
 - No interference after FAF

$T_j(\mathbf{X})$ = time point when acft j passes X
 ($X=0$ =FAF)

$$F_{X,j} = T_j(\mathbf{X}) - T_j(0)$$

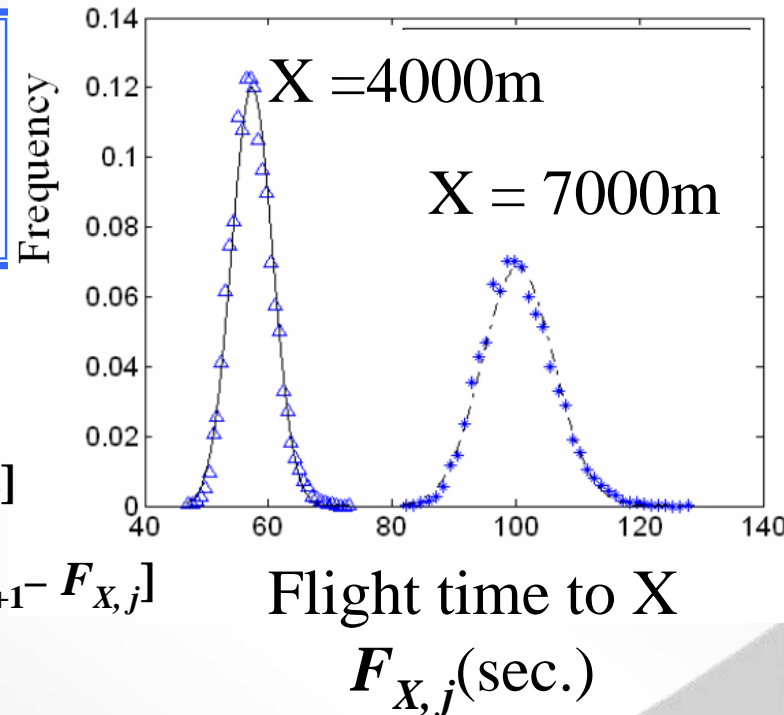
FAF

Flight time

$$T_{j+1}(\mathbf{X}) - T_j(\mathbf{X}) = (T_{j+1}(0) - T_j(0)) + (F_{X,j+1} - F_{X,j})$$

$$E[T_{j+1}(\mathbf{X}) - T_j(\mathbf{X})] = E[T_{j+1}(0) - T_j(0)] + E[F_{X,j+1} - F_{X,j}]$$

$$\text{Var}[T_{j+1}(\mathbf{X}) - T_j(\mathbf{X})] = \text{Var}[T_{j+1}(0) - T_j(0)] + \text{Var}[F_{X,j+1} - F_{X,j}]$$



Separation distribution at X :

$$a_t(j+1, X) = T_{j+1}(\mathbf{X}) - T_j(\mathbf{X})$$

$$\sim N(E[T_{j+1}(\mathbf{X}) - T_j(\mathbf{X})], \text{Var}[T_{j+1}(\mathbf{X}) - T_j(\mathbf{X})])$$

WV Encounter Risk

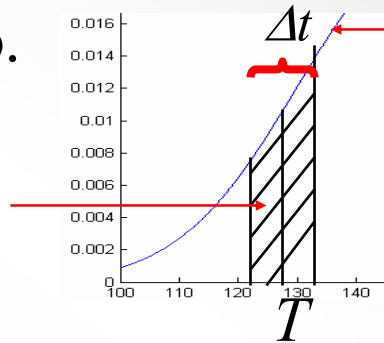


$$\underline{P(C|AB) * P(D|AB) * P(A) * P(B)}$$

Trailing acft passage prob.

$$P(A) =$$

$$P\left(|a_t(j+1, X) - T| \leq \frac{\Delta t}{2}\right)$$



PDF of $a_t(j+1, X)$

$$M(j, X) = \sum_{i=0}^n p(I(j, X, T) = 1) * \frac{\Delta t'}{\Delta t} \approx \int_{T_0}^{T_n} \frac{M(j, t, X)}{\Delta t} dt$$

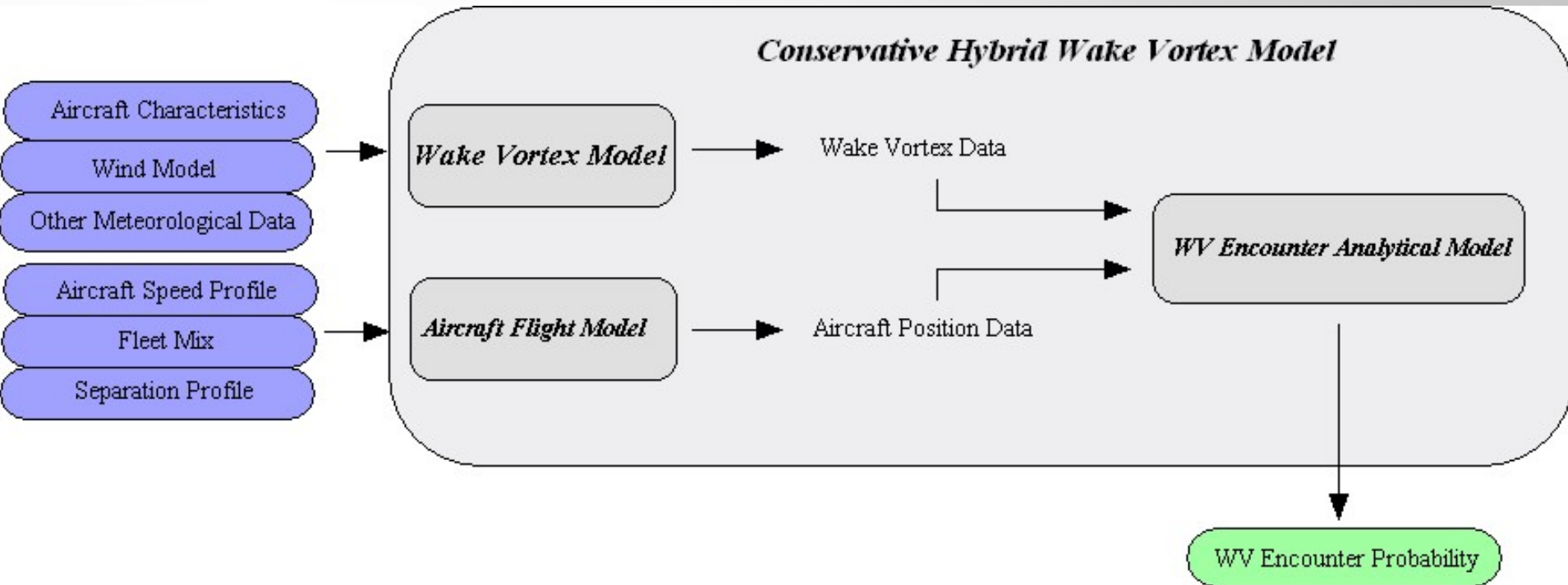
$\Delta t'$ is the simulation step, $\Delta t' \geq \Delta t$

$$M(j) = p(I(j, X_0) \cup I(j, X_1) \cup \dots \cup I(j, X_m)) \leq \int_{X_0}^{X_m} M(j, x) dx \equiv U(j)$$

WAVERET Toolbox



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Wake Vortex Encounter Risk Evaluation Toolbox (WAVERET)

Compared with Pure Simulation



- Pure Simulation
 - Must calculate each acft pos. and every WV
 - 250 Large acft/(sim.hour) by 2G Hz CPU, 1G memory
 - 40 hours (1.6 day) to have 10000 landings; 400 hours (16 days) to have 100000 landings
 - Risk for Large-Large mix is on 10^{-4} order
 - Time necessary to estimate risk varies by fleet mixes
- WAVERET Calculation
 - 2 hours to get the analytical calculation, independent of fleet mix
 - Risk for Large-Large mix is on 10^{-4} order